

Grade 12

Mathematics



MSI

Maths and Science Infinity

CAPS Syllabus

Number Patterns

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Activity 1

Given the quadratic number pattern: 244 ; 193 ; 148 ; 109 ; ...

- 1.1 Write down the next term of the sequence. (2)
- 1.2 Determine a formula for the n^{th} term of the pattern. (4)
- 1.3 Which term of the pattern will have a value of 508? (4)
- 1.4 Between which TWO consecutive terms of the quadratic pattern will the first difference be 453? (3)
- 1.5 Show that all the terms of the quadratic pattern are positive. (4)

[17]

Activity 2

Consider the following number pattern: -7 ; 0 ; 9 ; 20 ; ...

- 2.1 Show that the general term of this quadratic number pattern is given by
$$T_n = n^2 + 4n - 12.$$
 (4)
- 2.2 Which term of the sequence is equal to 128? (4)
- 2.3 Determine the general term of the first differences. (3)
- 2.4 Between which two terms of the quadratic pattern will the first difference be 599? (3)

[14]

Activity 3

A quadratic pattern has a second term equal to **6**, a third term equal to **2** and a fifth term equal to **-18**.

- 3.1 Calculate the second difference of the patter. (4)
- 3.2 Calculate the first term. (3)

[7]

Activity 4

The quadratic number pattern: 4 ; p ; 11 ; q ; 22 ; ...

- 4.1 Show that $p = 7$ and $q = 16$. (3)
- 4.2 Determine the general term, T_n , of the quadratic pattern. (4)
- 4.3 Determine the value of n if $T_n = 232$. (4)
- 4.4 If the sum of TWO consecutive terms in the pattern is 1 227.



Calculate the difference between these two terms. (5)

[16]

Activity 5

Given the sequence: 3; 9; 17; 27; ...

5.1 Write down the value of the next term of the sequence. (1)

5.2 Determine the expression n^{th} term of the sequence. (5)

5.3 Calculate the value of the first term that is greater than 269. (4)

[11]

Activity 6

Given the following quadratic number pattern: 5; -4; -19; -40; ...

6.1 Determine the second constant difference of the sequence. (2)

6.3 Determine the n^{th} term of the pattern. (4)

6.4 Which term of the sequence will be equal to -25 939? (3)

[9]

Activity 7

Given the quadratic sequence: 2; 3; 10; 23; ...

7.1 Write down the next term of the sequence. (1)

7.2 Determine the n^{th} of the sequence. (4)

7.3 Calculate the 20th term of the sequence. (2)

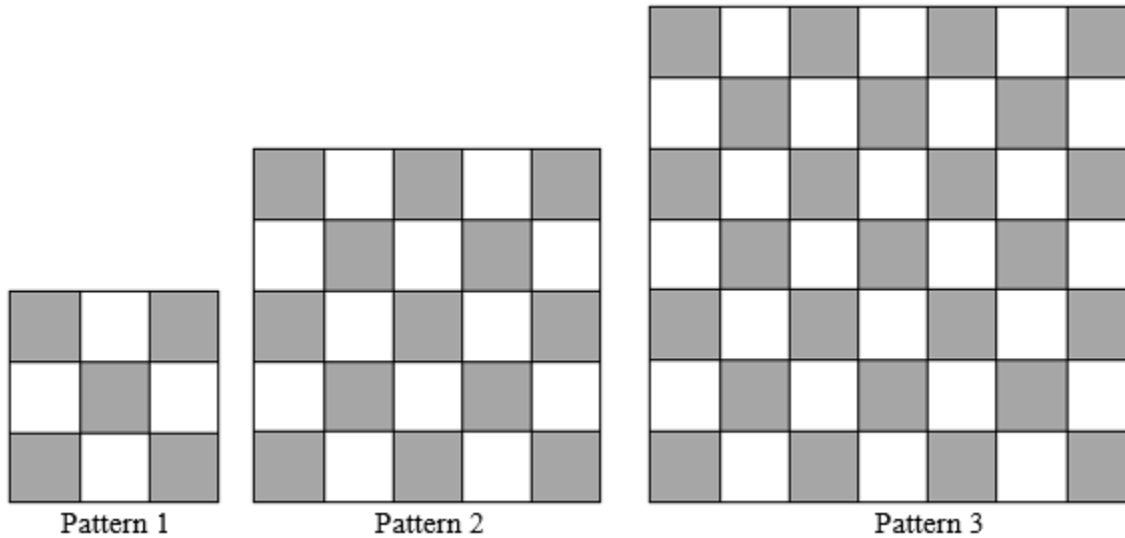
[7]



TUTORIAL 6

Activity 1

Grey and white squares are arranged into patterns as indicated below.



The total number of grey squares in the n^{th} pattern is given by $T_n = 2n^2 + 2n + 1$.

	Pattern 1	Pattern 2	Pattern 3
Number of grey squares	5	13	25

- 1.1 How many white squares will be in the FOURTH pattern? (2)
- 1.2 Determine the number of white squares in the 157th pattern. (3)
- 1.3 Calculate the largest value of n for which the pattern will have 613 grey squares. (4)
- 1.4 Show that the TOTAL number of squares in the n^{th} pattern is always an odd number. (3)

[12]

Activity 2

4 ; x ; y ; -11 ; ... are the first four terms of a quadratic number pattern.

$2p - 4$; $p - 3$; $-\frac{3}{4}p - 1$ are the first three terms of the same quadratic number pattern.

Calculate the values of p , x and y . [5]

Activity 3

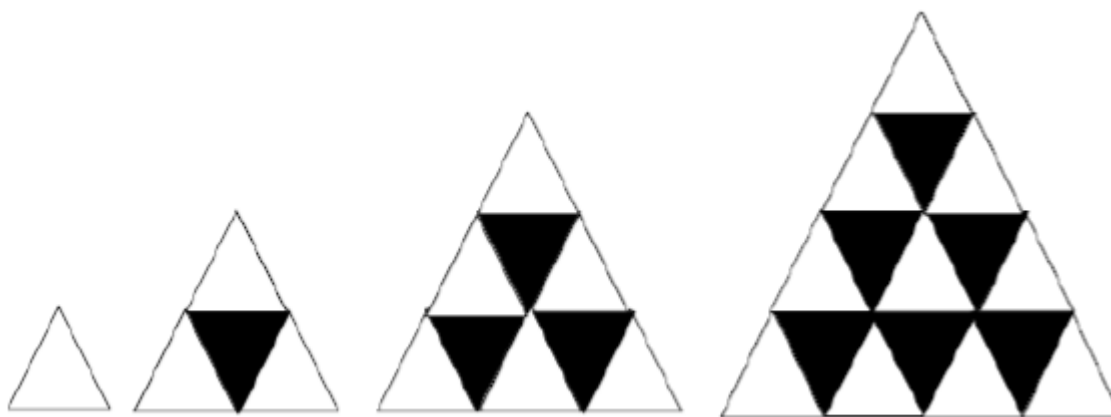
The number pattern 1 ; 5 ; 11 ; 19 ; ... is such that the second difference is constant.

- 3.1 Determine the 5th number in the pattern. (1)
- 3.2 Derive the formula for the n^{th} term of the pattern. (7)
- 3.3 What is the 100th number in the pattern. (3)

[11]

Activity 4

- 4.1 The general term of: 5; 12; 29; 48; 77; ... is $T_n = 3n^2 + 2$
Is this statement true? Show working to motivate your answer. (4)
- 4.2 The first four shapes of a sequence are shown below.



The table below shows the number of white and black triangles in the first three shapes.

Shape number, n	1	2	3	4	5
Number of white triangles	1	3	6		
Number of black triangles	0	1	3		
Total number of triangles	1	4	9		

- 4.2.1 Copy the table and complete it. (6)
- 4.2.2 How many triangles will there be altogether in the 12th shape? (2)
- 4.2.3 Determine the general term for the number of black triangles in the n^{th} shape. (7)
- 4.2.4 The number of black triangles in the n^{th} shape is 190. Determine the value of n . (5)

[24]

Activity 5

Consider the following shapes created with black and white tiles:



FIGURE 1



FIGURE 2



FIGURE 3

5.1 Complete the table:

Figure number	1	2	3	4		11
Number of shaded tiles	4		36		144	
Number of white tiles	1	9		49		
Total number of tiles	5		61	113		

(5)

5.2 Hence, determine a formula for the total number of tile in the n^{th} figure.

(3)

[8]

Activity 6

The following number pattern is given: 13 ; 27 ; 45 ; 67 ;

6.1 Is this a quadratic pattern? Justify your answer with relevant calculations. (2)

6.2 Determine the general term, T_n , of the quadratic number pattern. (4)

6.3 Calculate the value of T_{100} . (2)

6.4 The first difference between two consecutive terms of the quadratic number pattern is 110. Determine the values of these two terms. (5)

6.5 Show that ALL the terms of this quadratic number pattern will be odd numbers. (2)

[15]



TUTORIAL 7

Activity 1

Given the quadratic sequence: $p; 3; q; 19; \dots$

- 1.1 Calculate the value(s) of p and q if the second constant difference is 2. (4)
- 1.2 Determine the n^{th} term of the quadratic sequence. (4)
- 1.3 Determine the first term of the sequence will have a value greater than 10 301. (4)

[12]

Activity 2

2.1 Consider the sequence:

$-3; 2; 7; 12; \dots$

- 2.1.1 Write down the next two terms of the sequence. (2)
- 2.1.2 Determine the n^{th} term of the sequence. (2)
- 2.2 Determine the value of x if $1; 7; 9; x; 1; \dots$ is a quadratic sequence. (2)
- 2.3 $16; 33; 56; 85; \dots$ forms a quadratic sequence.
 - 2.3.1 Write down the next term in the pattern. (1)
 - 2.3.2 Determine the n^{th} term of the sequence. (4)
 - 2.3.3 Which term in the sequence has a value of 2080? (4)
 - 2.3.4 Determine an expression for the n^{th} term of the sequence of the first differences of this quadratic number pattern, and the 50th term. (5)
 - 2.3.5 $16; 33; 56; 85; \dots$ continues in the same as the above.
Write down the formula for the n^{th} term of the sequence. (2)

[22]

Activity 3

Consider the following sequence: $5; 12; 21; 32; \dots$

- 3.1 Write down the next TWO terms of the sequence. (1)
- 3.2 Determine the formula for the n^{th} term of the sequence. (5)

[6]

Activity 4

A given quadratic pattern $T_n = an^2 + bn + c$ has $T_2 = T_4 = 0$ and a second difference of 12. Determine the value of the 3rd term of the pattern. [6]



TUTORIAL 8

Activity 1

Consider the sequence: 6; 6; 2; -6 ; -18 ; ...

- 1.1 Write the next term of the sequence, if the sequence behaves consistently. (1)
- 1.2 Determine an expression the n^{th} term, T_n . (5)
- 1.3 Show that $-6\ 838$ is in the sequence. (4)

[10]

Activity 2

Consider the sequence: 8; 18; 30; 44; ...

- 2.1 Write down the next TWO terms of the sequence, if the patterns continue in the same way. (2)
- 2.2 Calculate the n^{th} term of the sequence. (6)
- 2.3 Which term in the sequence is 330? (4)

[12]

Activity 3

Consider the following sequence: 399; 360; 323; 288; 255; 224; ...

- 3.1 Determine the n^{th} term T_n in terms of n . (6)
- 3.2 Determine which term (or terms) has a value of 0. (3)
- 3.3 Which term in the sequence will have the lowest value? (1)

[10]

Activity 4

A quadratic pattern has a 2nd term equal to 1, a 3rd term equal -6 and a 5th term equal to -14 .

- 4.1 Calculate the second difference of this quadratic pattern. (5)
- 4.2 Hence, or otherwise, calculate the first term of the pattern. (2)

[7]



Activity 5

The sequence 3; 6; 11; 17; 24; ... is a quadratic sequence.

- 5.1 Write the next term. (1)
 - 5.2 Determine an expression for the n^{th} term T_n of the sequence. (4)
 - 5.3 What is the value of the first term of the sequence that is greater than 269? (4)
- [9]**

GEOMETRIC NUMBER PATTERNS

TUTORIAL 9

Activity 1

- 1.1 Consider the sequence: $\frac{1}{2}; 4; \frac{1}{4}; 7; \frac{1}{8}; 10; \dots$
 - 1.1.1 If the pattern continues in the same way, write down the next TWO terms in the sequence. (2)
 - 1.1.2 Calculate the sum of the first 50 terms of the sequence. (7)
- [9]**

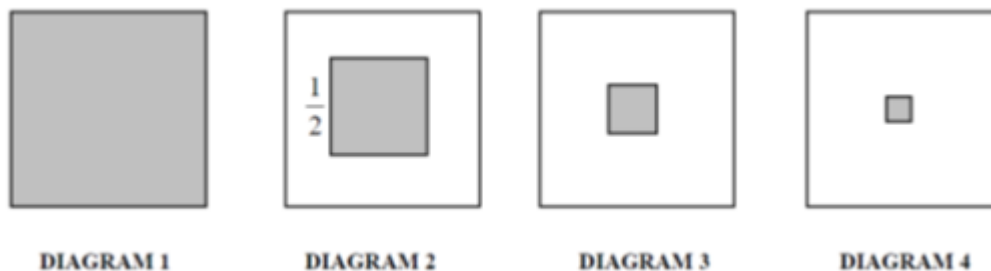
Activity 2

Given the geometric series: $8x^4 + 4x^3 + 2x^4 + \dots$

- 2.1 Determine the n^{th} term of the series. (1)
 - 2.2 For what value(s) of x will the series converge? (3)
 - 2.3 Calculate the sum of the series to infinity if $x = \frac{3}{2}$. (3)
- [7]**

Activity 3

A sequence of squares, each having side 1, is drawn as shown below. The first square is shaded, and the length of the side of each shaded square is half the length of the side of the shaded square in the previous diagram.



- 3.1 Determine the area of the unshaded region in DIAGRAM 3. (2)
- 3.2 What is the sum of the areas of the unshaded regions of the first seven squares? (5)
- [7]

Activity 4

Consider the series: $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots$

- 4.1 Express each of the following sums as a fraction of the form $\frac{a}{b}$:
- 4.1.1 The sum of the first two terms of the series. (1)
- 4.1.2 The sum of the first three terms of the series. (1)
- 4.1.3 The sum of the first four terms of the series. (1)
- 4.2 Make a conjecture about the sum of the first n terms of the given series. (2)
- 4.3 Use your conjecture to predict the value of the following:

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots + \frac{1}{2008 \times 2009} \quad (1)$$

[6]



Activity 5

- 5.1 Prove that $a + ar + ar^2 + \dots$ (to n terms) $= \frac{a(1-r^n)}{1-r}$ for $r \neq 1$. (6)
- 5.2 Given the geometric series $15 + 5 + \frac{5}{3} + \dots$
- 5.2.1 Explain why the series converges. (2)
- 5.2.2 Evaluate $\sum_{n=1}^{\infty} 5(3^{2-n})$ (3)
- 5.3 The sum of the first n terms of a sequence is given by $S_n = 2^{n+2} - 4$.
- 5.3.1 Determine the sum of the first 24 terms. (1)
- 5.3.2 Determine the 24th term. (2)
- 5.3.3 Prove that the n^{th} term of the sequence is 2^{n+1} . (3)

[17]

Activity 6

- 6.1 Prove that $a + ar + ar^2 + \dots$ (to n terms) $= \frac{a(1-r^n)}{1-r}$ for $r \neq 1$. (4)
- 6.2 Given the geometric series: $3 + 1 + \frac{1}{3} + \dots$
- Calculate the sum infinity. (3)

[7]

TUTORIAL 10

Activity 1

- 1.1 The first two terms of an infinite geometric sequence are 8 and $\frac{8}{\sqrt{2}}$. Prove, without the use of a calculator, that the sum of the series to infinity is $16 + 8\sqrt{2}$. (4)
- 1.2 The following geometric series is given: $x = 5 + 15 + 45 + \dots$ to 20 terms.
- 1.2.1 Write the series in sigma notation. (2)
- 1.2.2 Calculate the value of x . (3)

[9]



Activity 2

- 2.1 The sum of n terms of a sequence of number is given as: $S_n = \frac{n}{2}(5n + 9)$.
- 2.1.1 Calculate the sum to 23 terms of the sequence. (2)
- 2.1.2 Hence, calculate the 23rd term of the sequence. (3)
- 2.2 The first two terms of a geometric sequence and an arithmetic sequence are the same. The first term is 12. The sum of the first three terms of the geometric sequence is 3 more than the sum of the first three term of the arithmetic sequence.
- Determine TWO possible values for the common ratio, r of the geometric sequence. (6)
- [11]**

Activity 3

The following sequence is a combination of an arithmetic and geometric sequence:

$$3; 3; 9; 6; 15; 12; \dots$$

- 3.1 Write down the next TWO terms. (2)
- 3.2 Calculate $T_{52} - T_{51}$. (5)
- 3.3 Prove that ALL the terms of this infinite sequence will be divisible by 3. (2)
- [9]**

Activity 4

- 4.1 Consider the geometric sequence: $4 - 2 + 1 - \dots$
- 4.1.1 Determine the next term of the sequence. (2)
- 4.1.2 Determine n if the n^{th} term is $\frac{1}{64}$. (4)
- 4.1.3 Calculate the sum to infinity of the series $4 - 2 + 1 - \dots$ (2)
- 4.2 If x is a REAL number, show that the following sequence can NOT be geometric:
- $$1; x + 1; x - 3; \dots \quad (4)$$
- [12]**



Activity 5

An athlete runs along a straight road. His distance d from a fixed-point P on the road is measured at different times, n , and has the form $d(n) = an^2 + bn + c$. The distances are recorded in the table below.

Time (in seconds)	1	2	3	4	5	6
Distance (in metres)	17	10	5	2	r	s

- 5.1 Determine the values of r and s . (3)
 - 5.2 Determine the values of a , b and c . (4)
 - 5.3 How far is the from P when $n = 8$? (2)
 - 5.4 Show that the athlete is moving towards when $n < 5$, and away from P when $n > 5$. (4)
- [13]**

Activity 6

Given the geometric sequence: 27; 9; 3; ...

- 6.1 Determine the formula for T_n , the n^{th} term of the sequences. (2)
 - 6.2 Why does the sum to infinity for this sequence exist? (1)
 - 6.3 Determine S_∞ . (2)
- [5]**

TUTORIAL 11

Activity 1

- 1.1 Given the sequence: 4; x ; 32
Determine the value(s) of x if the sequence is:
 - 1.1.1 Arithmetic (2)
 - 1.1.2 Geometric (3)
- 1.2 Determine the value of P if $P = \sum_{k=1}^{13} 5 \cdot 3^k$ (4)
- 1.3 Prove that for any arithmetic sequence of which the first term is a and the constant difference is d , the sum to n terms can be expressed as $S_n = \frac{n}{2} [2a + (n - 1)d]$. (4)

[13]



Activity 2

- 2.1 Twenty water tanks are decreasing in size such that the volume of each tank is half the volume of the previous tank. The first tank is empty, but the other 19 tanks full of water.



Would it be possible for the first water tank to hold the water from the other 19 tanks?
Motivate your answer. [4]

Activity 3

- 3.1 1; 3; 5 are the first three terms of the first difference of a quadratic sequence.
The 7th term of the quadratic sequence is 35.

3.1.1 Determine the 6th and 5th terms of the quadratic sequence. (4)

3.1.2 Determine the n^{th} term of the quadratic sequence. (5)

- 3.2 Calculate the value of n if:

$$\sum_{k=1}^n 2(3)^{k-1} = 531\,440 \quad (5)$$

[14]

