

MATHEMATICS

MATERIAL FOR GRADE 12

FUNCTIONS

MEMORANDA

QUESTION 1

1.1.1	$x = -2 ✓$ & $y = -3 ✓$	✓✓ Answers (2)
1.1.2.1	$(2) \left(-\frac{5}{2} + 3 \right) = t ✓$ $2 \left(\frac{1}{2} \right) = t$ $t = 1 ✓$	✓ Substituting $\left(0; -\frac{5}{2} \right)$ ✓ $t = 1$ (2)
1.1.2.2	$p = 2 ✓, (2)^2 + q = 3 ✓$ $q = -1 ✓$	✓ $p = 2$ ✓ Substitution ✓ $q = -1$ (3)
1.2	$(x + 2)(y + 3) = 1 ✓$ $y + 3 = \frac{1}{x+2} ✓$ $y = \frac{1}{x+2} - 3 ✓$	✓ Substituting 1 ✓ Simplification ✓ Answer (3)
1.3	$x - 1 = \frac{1}{x+2} - 3$ $x + 2 = \frac{1}{x+2}$ $(x + 2)^2 = 1$ $x + 2 = \pm 1$ $x = -1 \text{ or } x = -3$ $y = -2 \text{ or } y = -4$ $(-1; -2) (-3; -4)$	✓ Equating ✓ Simplification ✓ factors ✓ x-values ✓✓ y-values (6)
1.4.1	$k < -1$	✓✓ Answer (2)
1.4.2	$-1 < k < 3$	✓ -1 & 3 ✓ Notation (2)
1.5.1	$D: x \in \mathbf{R} ✓, x \neq -2 ✓$ or $x \in (-\infty; \infty), x \neq -2$ or $x < -2$ or $x > 2$	✓ $x \in \mathbf{R}$ ✓ $x \neq -2$ (2)
1.5.2	$R: y \geq -1 ✓, y \in \mathbf{R}$ or/of $y \in [-1; \infty)$	✓ Answer (1)
		[22]

QUESTION 2

2.1.1	$v(x)$ is not a function, ✓ there are two different y –values for each x . A vertical line test fails: Line cuts the graph at more than one point. ✓	✓ Answer ✓ Reason (2)
2.1.2	$y \in \{(-\infty; 0] \cup [0; \infty)\}$ ✓✓ or $y \geq 0$ or $y \leq 0$	✓✓ Ans (2)
2.1.3.1	$y < 0$ ✓	✓ Answer (1)
2.1.3.2	$0 < x \leq \frac{49}{100}$ ✓	✓ Notation ✓ boundaries (2)
2.1.4	$y = x^2$ ✓ $x \in [0; \infty)$ ✓ $y = x^2$ $x \in (-\infty; 0]$ ✓	✓ $y = x^2$ ✓ $x \in [0; \infty)$ ✓ $y = x^2$ $x \in (-\infty; 0]$ (3)
2.1.5	$y \in (0; 1)$ or $0 < y < 1$ $(-1; 0)$ or $-1 < y < 0$	✓ Answer (1)
		[11]

QUESTION 3

3.1	At C ; $x = -\frac{b}{2a}$ $= -\frac{-2}{2(-1)} = -1$ $y = -(-1)^2 - 2(-1) + 3 = 4$ coordinated of C are $(-1; 4)$	✓ ✓ ✓	$x = -\frac{-2}{2(-1)}$ x value y value	(3)
3.2	$y = 0$ $-x^2 - 2x + 3 = 0$ $x^2 + 2x - 3 = 0$ $(x - 1)(x - 3) = 0$ $x = 1$ or $x = -3$ Coordinates are B(1; 0) and A(-3 ; 0)	✓ ✓ ✓	$y = 0$ $(x - 1)(x - 3) = 0$ x – values	(3)
3.3	(0; 3)	✓	answer	(1)
3.4	equation of g $y - 0 = \frac{4-0}{-1-(-3)} (x - (-3))$ $y = 2(x + 3)$ $y = 2x + 6$	✓ ✓ ✓	substitution value of m equation of g	

		E is (0; 6) C(-1; 4) $CE = \sqrt{(0 - (-1))^2 + (6 - 4)^2}$ $= \sqrt{5}$	✓ ✓ ✓	E is (0; 6) substitution in the distance formula. answer	(6)
3.5		$x = 2y + 6$ $y = \frac{x}{2} - 3 = \frac{1}{2}x - 3$ accept also $y = mx + c$, so inverse is $x = my + c$ $y = \frac{x}{m} - c$	✓ ✓	interchange x and y answer	(2)
					[15]

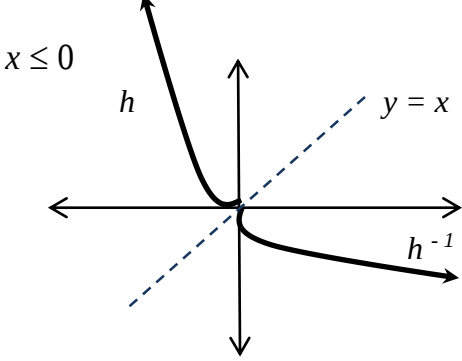
QUESTION 4

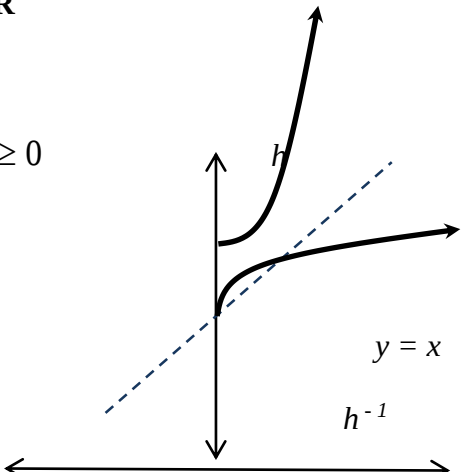
4.1		$e = 2$ $q = 2$	✓ ✓	$e = 2$ $q = 2$	(2)
4.2		$g(x) = 2^x + 2$ y -int.; let $x = 0$ $g(0) = 2^0 + 2 = 3$ so both graphs pass at (0; 3) $3 = \frac{3}{0-p} + 2$ $\frac{3}{0-p} = 1$ $-p = 3 \quad \therefore p = -3$	✓ ✓ ✓	(0; 3) or y - int. subst. (0;3) $p = -3$	(3)
4.3		at A: $x = -3$ $y = 2^{-3} + 2$ $= 2 \frac{1}{8}$ A(-3; $2 \frac{1}{8}$) or A(-3; 2,125)	✓ ✓ ✓	at A : $x = -3$ subst. answer	(3)

4.4		$-3 < x \leq 0$	✓ ✓	$x > -3$ $x \leq 0$	(2)
4.5		$f(x) = \frac{3}{x+3} + 2$ after shifting $f(x-2) = \frac{3}{x+3-2} + 2$ $f(x-2) = \frac{3}{x+1} + 2$	✓ ✓	Subst. x by $x-2$ answer	(2)
					[12]

QUESTION 5

5.1	$x = -3$ $y = 2$	$\checkmark x = -3$ $\checkmark y = 2$ (2)
5.2	$y = \frac{a}{x+p} + q$ $= \frac{a}{x+3} + 2$ $0 = \frac{a}{-1+3} + 2$ $-2 = \frac{a}{2}$ $-4 = a$ $\therefore f(x) = \frac{-4}{x+3} + 2$	$\checkmark$ substitution of p and q $\checkmark$ substitution of $(-1; 0)$ $\checkmark a = -4$ (3)
5.3	$y = -(x+3) + 2$ $= -x - 3 + 2$ $= -x - 1$ OR $y = -x + c$ $2 = -(-3) + c$ $c = -1$ $y = -x - 1$	$\checkmark$ substitution $\checkmark$ answer (2) $\checkmark$ substitution $\checkmark$ answer (2)
5.4	$x \in \mathbb{R}; x \neq -2$	$\checkmark x \in \mathbb{R}$ $\checkmark x \neq -2$ (2)
5.5	$k(x) = \frac{-4}{-x+3} + 2$ $= \frac{-4}{-(x-3)} + 2$ $= \frac{4}{x-3} + 2$	$\checkmark\checkmark$ answer (2)
5.6	$-3 < x \leq -1 \text{ or } x \geq 0$ OR $x \in (-3; -1] \text{ or } x \in [0; \infty)$	$\checkmark -3 < x \leq -1$ $\checkmark x \geq 0$ (2) $\checkmark x \in (-3; -1]$ $\checkmark x \in [0; \infty)$ (2)

5.7	$g(x) = bx^2 - 2$ $2 = b(-3)^2 - 2$ $4 = b(9)$ $b = \frac{4}{9}$ $\therefore g(x) = \frac{4}{9}x^2 - 2$	$\checkmark c = -2$ $\checkmark$ substitution $(-3; 2)$ $\checkmark b = \frac{4}{9}$ (3)
5.8	$h: y = \frac{4}{9}x^2$ $h^{-1}: x = \frac{4}{9}y^2$ $y^2 = \frac{9}{4}x$ $y = \pm \sqrt{\frac{9}{4}x}; x \geq 0$ $y = \pm \frac{3}{2}\sqrt{x}; x \geq 0$	$\checkmark h(x) = \frac{4}{9}x^2$ $\checkmark$ swop x and y $\checkmark$ answer with restriction (3)
5.9	$x \leq 0$ 	$\checkmark$ form of h $\checkmark$ form of h^{-1} (must fit form of h) (2) $\checkmark$ form of h $\checkmark$ form of h^{-1} (must fit form of h) (2)

	OR $x \geq 0$ 	[21]
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QUESTION 6

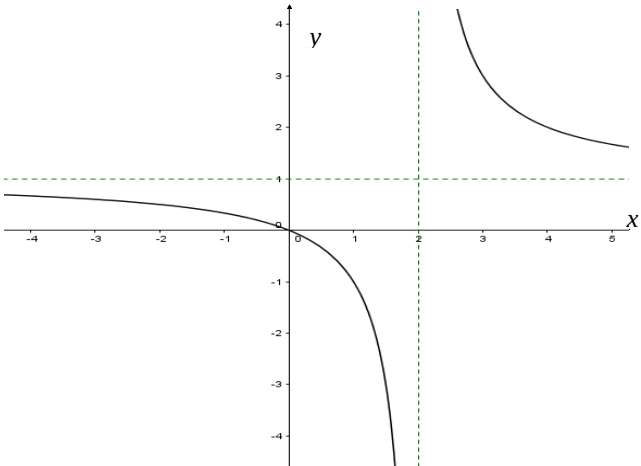
6.1	$x = 0:$ $y = -(0 + 1)^2 + 9$ $= -1 + 9$ $= 8$ $\therefore C(0; 8)$	✓ x-coordinate of C ✓ y-coordinate of C (2)
6.2	$q = 8$ (Horizontal asymptote) $y = a \cdot 2^x + 8$ Turning point: $D(-1; 9)$ $9 = a \cdot 2^{-1} + 8$ $1 = \frac{a}{2}$ $2 = a$ $\therefore g(x) = 2 \cdot 2^x + 8$ $= 2^{x+1} + 8$	✓ $q = 8$ ✓ substitution of $D(-1; 9)$ ✓ $a = 2$ (3)
6.3	$y > 8$ OR $y \in (8; \infty)$	✓ notation

		✓ answer (2)
6.4	D'(-1; 7)	✓ answer (1)
6.5	Reflection about the x-axis, and a translation of 1 unit left and 18 units up. OR Reflection about the line $y = 9$ and a translation of 1 unit left.	✓ Reflection x-axis ✓ 1-unit left ✓ 18 units up (3) ✓✓ Reflection $y = 9$ ✓ 1-unit left (3)
6.6	$y = \log_{\frac{1}{3}} x$ OR $y = -\log_3 x$ OR $y = \log_3 \frac{1}{x}$	✓ answer (1) ✓ answer (1) ✓ answer (1)
6.7	$y = \left(\frac{1}{3}\right)^x$ is a decreasing function $\therefore$ the bigger the x-value the smaller the y-value maximum value of $f = 9$ minimum value: $y = \left(\frac{1}{3}\right)^{9-5}$ $= \left(\frac{1}{3}\right)^4$ $= \frac{1}{81}$ OR $y = 3^{-(f(x)-5)}$ $= 3^{-f(x)+5}$ $= 3^{(x+1)^2-9+5}$ $= 3^{(x+1)^2-4}$ $\therefore$ minimum $= 3^{-4}$ $= \frac{1}{81}$	✓ substitution of 9 ✓ answer $\left(\frac{1}{3}\right)^4$ or $\frac{1}{81}$ Accept 0,01 (2) ✓ substitution of $f(x)$ ✓ answer $\left(\frac{1}{3}\right)^4$ or $\frac{1}{81}$ Accept 0,01 (2) (2) [14]

QUESTION 7

7.1	$R(-2; 4)$	✓ -2 ✓ 4	(2)
7.2	B (-4; 0) through symmetry $\therefore AB = 4$ units OR roots: $(x+2)^2 = 4$ OR $-x^2 - 4x = 0$ $\therefore x + 2 = \pm 2$ $x(x+4) = 0$ $\therefore x = 0$ or -4 $\therefore AB = 4$ units	✓ -4 ✓ 4 units	(2)
7.3	$m = -2$ eqn: $y = -2x$	✓ $m = -2$ ✓ eqn	(2)
7.4	$x < -2$ OR $x > 0$	✓ $x < -2$ ✓ $x > 0$	(2)
7.5	$h(x) = f(-x) = -(-x + 2)^2 + 4$ sym- axis: $x = 2$	✓ $h(x)$ ✓ $x = 2$ Answer Only = FULL marks	(2)
7.6	$p(x) = -f(x) = (x + 2)^2 - 4$ range: $y \geq -4 ; y \in R$ OR $[-4; \infty)$	✓ $p(x)$ ✓ $y \geq -4$ Answer Only = FULL marks	(2)
			[12]

QUESTION 8

8.1	$x = 2$; $y = 1$	$\checkmark x = 2$ $\checkmark y = 1$	(2)
8.2	y-int: $y = 0$; x-int: $\frac{2}{x-2} = -1$ $\therefore x - 2 = -2$ $\therefore x = 0$	$\checkmark y = 0$ $\checkmark \frac{2}{x-2} = -1$ $\checkmark x = 0$	(3)
8.3		$\checkmark$ asymptotes $\checkmark$ x/y intercept $\checkmark$ shape	(3)
8.4	$x \in R; x \neq 2$	$\checkmark x \in R$ $\checkmark x \neq 2$	(2)
8.5.1	Graph shifts(translates) 3 units to the left	$\checkmark\checkmark$ 3 units to the left	(2)
8.5.2	Graph shifts(translates) 2 units down	$\checkmark\checkmark$ 2 units down	(2)
			[14]

QUESTION 9

9.1.1	$A(0; 1)$	✓ Answer	(1)
9.1.2	$f^{-1}: y = \log_3 x$	✓ $y = \log_3 x$	(1)
9.1.3	$0 < x \leq 1$	✓ endpoints ✓ notation	(2)
9.1.4	$y = 0$	✓ $y = 0$	(1)
9.2.1	$f(x) = \sqrt{\frac{x}{a}}$ OR $f^{-1}(x) = ax^2$ $(8; 2): 2 = \sqrt{\frac{8}{a}} \Rightarrow a = 2$ $(2; 8): 8 = a(2)^2$ $\Rightarrow a = 2$ $\therefore f(x) = \sqrt{\frac{x}{2}}$ $\therefore$ $f^{-1}(x) = 2x^2$ $f(x): x = 2y^2$ $y = \sqrt{\frac{x}{2}}$	✓ $a = 2$ ✓ eqn	(2)
9.2.2	$f^{-1}(x) = ax^2$ $(2; 8): 8 = a(2)^2 \Rightarrow a = 2$ $\therefore f^{-1}(x) = 2x^2$	✓ eqn	(1)
9.2.3	$(-3; -1)$	✓ each value	(2)
			[10]

QUESTION 10

10.1	$x = 2$ and $y = 1$	$\checkmark x = 1 \checkmark y = 1$ (2)
10.2	$\frac{3}{x-2} = -1$ $3 = -1(x-2)$ $3 = -x + 2$ $3 - 2 = -x$ $-1 = x$	$\checkmark x = -1$ $\checkmark y = 0$ (2)
10.3	$g(x) = \frac{3}{x+1-2} + 1$ $g(x) = \frac{3}{x-1} + 1$ $x \in \mathbb{R}, x \neq 1$ OR $(-\infty; 1) \text{ or } (1; \infty)$	$\checkmark g(x) = \frac{3}{x-1} + 1$ $\checkmark \checkmark x \in \mathbb{R}, x \neq 1$ or $\checkmark (-\infty; 1) \text{ or } \checkmark (1; \infty)$ (3)
10.4	$y = x + p + q$ $= x - 2 + 1$ $= x - 1$ OR $y = x + c$ $1 = 2 + c$ $1 - 2 = c$ $y = x - 1$ (2;1)	$\checkmark x$ $\checkmark -1$ (2)
		[9]

QUESTION 11

11.1	$y = 2x^2$ $k: x = 2y^2$ $y = \sqrt{\frac{x}{2}} ; y \geq 0$	✓ interchange x and y ✓ $y = \sqrt{\frac{x}{2}}$ (2)
11.2		h ✓ shape ✓ y-int. ✓ asymptote. k ✓ shape ✓ y-intercept (5)
11.3	$(0; \infty)$ OR $y > 0; y \in \mathbb{R}$	✓ $(0; \infty)$ (1)
11.4	$0 < x \leq 0,57$ OR $x \in (0; 0,57]$	✓ $0 < x$ ✓ $x \leq 0,57$ (2)
11.5	$t > 1$, OR $t \in (1; \infty)$	✓✓ answer (2)
		[12]

QUESTION 12

12.1	$y = 6 - 0$ $y = 6$	$\checkmark y = 6$ (1)
12.2	$h(x) = \frac{4}{x-1}$	$\checkmark h(x) = \frac{4}{x-1}$ (1)
12.3	$\frac{4}{x-1} = 6 - x$ $4 = (6 - x)(x - 1)$ $= 6x - x^2 - 6 + x$ $0 = -x^2 + 7x - 6 - 4$ $= -x^2 + 7x - 10$ $x^2 - 7x + 10 = 0$ $(x - 5)(x - 2) = 0$ $x = 5 \text{ or } x = 2$	$\checkmark \frac{4}{x+1} = 6 - x$ $\checkmark$ standard form $\checkmark$ both answers (3)
12.4	$CD = g(x) - f(x)$ $= 6 - x - \frac{4}{x}$	$\checkmark g(x) - f(x)$ $\checkmark 6 - x - \frac{4}{x}$ (2)
12.5	$\frac{dCD}{dx} = 0$ $6 - x - 4x^{-1}$ $-1 + \frac{4}{x^2} = 0$ $x^2 = 4$ $x = -2 \text{ or } x = 2$ $\therefore x = 2$	$\checkmark = 0$ $\checkmark -1 + \frac{4}{x^2}$ $\checkmark x^2 = 4$ $\checkmark x = 2$ (4)
		[11]

QUESTION 13

13.1	$p = -3$ $q = -2$	A✓ p value A✓ q value	(2)
13.2	$h(x) = \frac{a}{x+p} + q$. P(4;-4) is a point on h $-4 = \frac{a}{4-3} - 2$ $\therefore a = -2$	CA✓ subst. p , q and point P CA(negative)✓ a value	(2)
13.3	$h(x) = \frac{-2}{x-3} - 2$ $h(0) = \frac{-2}{0-3} - 2$ $= -1\frac{1}{3}$ $\left(0; -1\frac{1}{3}\right)$	A✓ substituting $x = 0$ CA(negative)✓ answer	(2)
13.4	$x = 1$	CACA✓✓ answer	(2)
13.5	$y = -(x+p) + q$ $y = -(x-3) - 2$ $y = -x + 1$ $\therefore c = 1$	CA✓ substitution of p and q values into equation of line of symmetry CA✓ answer	(2)

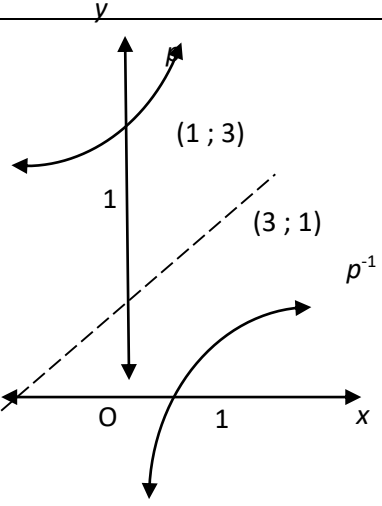
	OR $y = -x + c$ <i>Point of intersection of asymptotes</i> (3 ; -2) $-2 = -3 + c$ $c = 1$	CA✓substitution of (3 ; -2) into equation of line of symmetry CA✓answer	(2)
			[10]

QUESTION 14

14.1	D(0 ; -10)	A(must be in coordinate form)✓ answer	(1)
14.2	$x^2 - 3x - 10 = 0$ $(x + 2)(x - 5) = 0$ $x = -2$ or $x = 5$ A(-2;0) B(5;0)	A✓ $x^2 - 3x - 10 = 0$ CA✓factors CACA(negative and positive)✓✓ each x – value A(-2;0) B(5;0)	(4)
14.3	$a = 2$ and $q = -10$	CA(positive)✓a – value A✓ q – value	(2)
14.4	$x = -\frac{b}{2a} = -\frac{(-3)}{2(1)} = \frac{3}{2}$ or $x = \frac{-2+5}{2} = \frac{3}{2}$ $y = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 10 = -\frac{49}{4} / -12,25 / -12\frac{1}{4}$ C(1,5 ; -12,25)	A✓ $x = -\frac{b}{2a} = -\frac{(-3)}{2(1)} = \frac{3}{2}$ or	

		$CA \checkmark x = \frac{-2+5}{2} = \frac{3}{2}$ CA ✓ substitution CA ✓ minimum value	(3)
14.5	(−1,5 ; −9,25)	CA ✓ x – value CA ✓ y – value	(2)
14.6	$x \geq \frac{3}{2}$ OR $g'(x).h'(x) \geq 0$ $(2x-3).2 \geq 0$ $x \geq \frac{3}{2}$	CACA ✓ ✓ answer CA ✓ product of derivatives CA(positive) ✓ answer penalize 1 mark for incorrect notation	(2) (2)
			[14]

QUESTION 15

15.1	$y = \log_3 x$	AA✓✓ answer	(2)
15.2		A✓ Shape of p and p^{-1} A✓ y – intercept of p A✓ x – intercept of p^{-1} A✓ point on each graph	(4)
15.3	$\log_3 x = 3$ $x = 27$ $0 < x \leq 27$	M✓ setting up equation CA✓ $x = 27$ CACA✓✓ for end points and inequality ANSWER ONLY full marks	(4)
			[10]

QUESTION 16

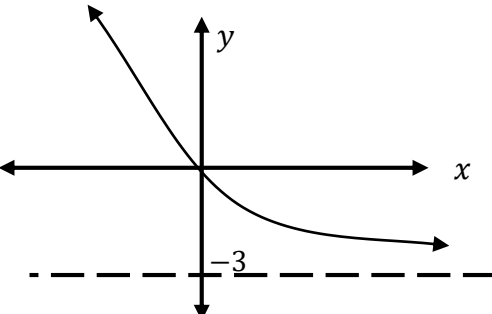
16.1		$y = 2x - 12$ $m = 2$	✓	$m = 2$	(1)
16.2		$(x - 4)(x + 2) = 2x - 12$ $x^2 - 2x - 8 - 2x + 12 = 0$ $x^2 - 4x + 4 = 0$ $(x - 2)^2 = 0$ $x = 2$ $y = 2(2) - 12$ $= -8$ $A(2; -8)$ OR $y = (x - 4)(x + 2)$ $= x^2 - 2x - 8$ $\frac{dy}{dx} = 2x - 2$ since g is a tangent passing through A <i>i. e.</i> $2x - 2 = 2$ $x = 2$ $y = 2(2) - 12$ $= -8$ $A(2; -8)$	✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓	$(x - 4)(x + 2)$ $= 2x - 12$ standard form factors $x = 2$ $y = 8$ OR $y = x^2 - 2x - 8$ $\frac{dy}{dx} = 2x - 2$ $2x - 2 = 2$ $x = 2$ $y = -8$	(5)
16.3		$h(x) = f(-x)$ $= (-x - 4)(-x + 2)$ $= (x + 4)(x - 2)$ OR $h(x) = x^2 + 2x - 8$	✓ ✓	subst x by $-x$ answer	(2)
16.4		$(x - 4)(x + 2)(2x - 12) < 0.$ $4 < x < 6$	✓ ✓	$x < 6$ $x > 4$	(2)
16.5		$x = 2y - 12$ $y = \frac{1}{2}x + 6$	✓ ✓	interchange x and y $y = \frac{1}{2}x + 6$	(2)
					[12]

QUESTION 17

17.1		$x = -1$ $y = (-1)^2 + 2(-1) - 3$ $= -4$	✓ ✓ ✓	$x = -1$ $y = (-1)^2 + 2(-1) - 3$ $y = -4$	(3)
17.2		$A(0; -3)$	✓	Answer	(1)
17.3		$y = \frac{a}{x+1} - 4$ $-3 = \frac{a}{0+1} - 4$ $1 = a$ $g(x) = \frac{1}{x+1} - 4$	✓ ✓ ✓ ✓	subst. of p subst. of q subst (0; -3) answer	(4)
17.4		Before shifting, the points were at (1; 1) and (-1; -1) applying the shift on these points yields: (1 - 1; 1 - 4) and (-1 - 1; -1 - 4) so the points are: (0; -3) and (-2; -5) OR from the point of intersection of asymptotes (-1; -4) we move a unit up and to the right or a unit down and to the left since $a = 1$ so the points are: (-1 + 1; -4 + 1) and /or (-1 - 1; -4 - 1) the points are (0; -3) and (-2; -5)	✓ ✓ ✓ ✓ OR ✓ ✓ ✓ ✓	(1 - 1; 1 - 4) (-1 - 1; -1 - 4) (0; -3) (-2; -5) OR (-1 + 1; -4 + 1) (-1 - 1; -4 - 1) (0; -3) (-2; -5)	(4)

		OR equation of the line of symmetry is $y = (x + 1) - 4$ $y = x - 3$ $x - 3 = \frac{1}{x + 1} - 4$ $x + 1 = \frac{1}{x + 1}$ $x^2 + 2x + 1 = 1$ $x^2 + 2x = 0$ $x = 0 \text{ or } x = -2$ $y = -3 \text{ or } y = -5$ The points are $(0; -3)$ and $(-2; -5)$	OR ✓ ✓ ✓ ✓	OR equation of the line of symmetry equating the two functions values of x values of y	
17.5		$y \leq 4; y \in \mathbb{R}$	✓✓	answer	(2)
					[14]

QUESTION 18

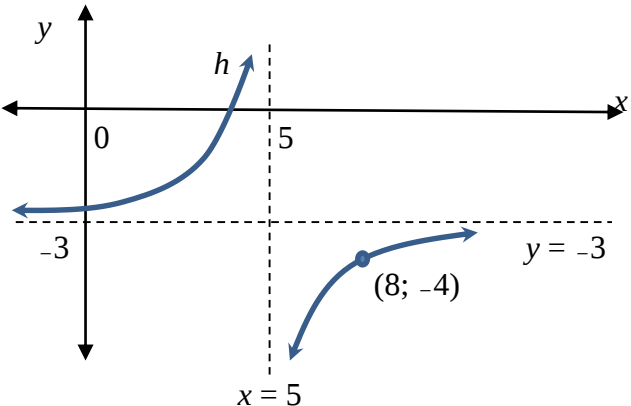
18.1			✓ ✓ ✓	shape graph passing through the origin asymptote	(3)
18.2		$5 = 3^{-x+1} - 3$ $8 = 3^{-x+1}$ $(-x + 1)\log 3 = \log 8$ $(-x + 1) = \frac{\log 8}{\log 3}$ $x = 1 - \frac{\log 8}{\log 3}$ $= -0,89$	✓ ✓ ✓	substitution applications of log answer	(3)
18.3		graph was reflected about the y -axis graph was shifted 3 units up graph was shifted one unit to the left.	✓ ✓ ✓	reflect about y -axis shift 3 units up one unit to the left	(3)

		OR graph was shifted 3 units up graph was reflected about the y axis graph was shifted to one the left. NB reflection should not be mentioned after horizontal shift	OR ✓ ✓ ✓	OR shift 3 units up reflect about y –axis one unit to the left	
					[9]

QUESTION 19

19.1	$y = \frac{-3}{0-2} + 1$ $= \frac{3}{2} + 1$ $= 2,5 \quad \text{or} \quad y = \frac{5}{2}$ $\therefore (0; 2,5) \quad \text{or} \quad \left(0; \frac{5}{2}\right)$	✓ $x = 0$ ✓ $y = 2,5$ or $y = \frac{5}{2}$ (2)
19.2	$0 = \frac{-3}{x-2} + 1$ $-1 = \frac{-3}{x-2}$ $x - 2 = 3$ $x = 5$ $\therefore (5; 0)$	✓ $y = 0$ ✓ $x = 5$ (2)

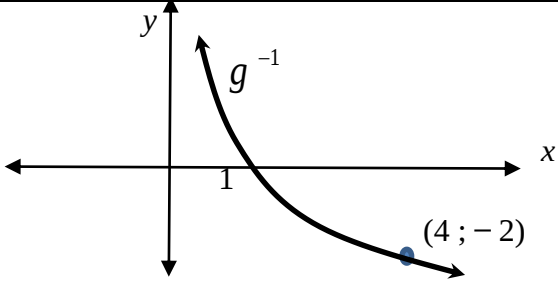
19.3		✓ shape ✓ both intercepts ✓ both asymptotes (3)

19.4	$y \in \mathbb{R}; y \neq 1$ OR $y < 1 \quad \text{or} \quad y > 1$ OR $y \in (-\infty; 1) \cup y \in (1; \infty)$	$\checkmark y \in \mathbb{R}$ $\checkmark y \neq 1 \quad (2)$ $\checkmark y < 1; \quad \checkmark y > 1 \quad (2)$ $\checkmark y \in (-\infty; 1)$ $\checkmark y \in (1; \infty) \quad (2)$
19.5	$h(x) = \frac{-3}{x-5} - 3$	$\checkmark \frac{-3}{x-5}$ $\checkmark -3 \quad (2)$
19.6	From the graph of h:  $5 < x \leq 8 \quad \text{or} \quad x \in (5; 8]$ OR From translations: $h(x) \leq -4 \quad \therefore f(x) \leq 0$ (4 units up) If $f(x) \leq 0$, then $2 < x \leq 5$ $\therefore$ for $h(x)$: $5 < x \leq 8$ (3 units to the right)	$\checkmark (8; -4)$ $\checkmark 5 < x$ $\checkmark x \leq 8 \quad (3)$ $\checkmark f(x) \leq 0$ $\checkmark f(x): 2 < x \leq 5$ $\checkmark h(x): 5 < x \leq 8 \quad (3)$
19.7	$k(x) = \frac{3x-5}{x-1}$ By dividing $x-1$ into $3x-5$: $k(x) = \frac{-2}{x-1} + 3$ $\therefore$ The asymptotes are: $x=1$ and $y=3$	$\checkmark k(x) = \frac{-2}{x-1} + 3$ $\checkmark x=1$ $\checkmark y=3 \quad (3)$

	OR $k(x) = \frac{3x - 5}{x - 1}$ $k(x) = \frac{3(x - 1) - 2}{x - 1}$ $k(x) = 3 - \frac{2}{x - 1}$ $\therefore$ The asymptotes are: $x = 1$ and $y = 3$	$\checkmark k(x) = \frac{-2}{x - 1} + 3$ $\checkmark x = 1$ $\checkmark y = 3$ (3) [17]
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QUESTION 20

20.1	$x = -1$	$\checkmark x = -1$ (1)
20.2	$(-1; -8)$	$\checkmark$ answer (1)
20.3	$2(x + 1)^2 - 8 = 0$ $2(x + 1)^2 = 8$ $(x + 1)^2 = 4$ $x + 1 = \pm 2$ $x = 1$ of $x = -3$ $\therefore PQ = 1 + 3 = 4$ units OR $2(x^2 + 2x + 1) - 8 = 0$ $2x^2 + 4x + 2 - 8 = 0$ $2x^2 + 4x - 6 = 0$ $x^2 + 2x - 3 = 0$ $(x + 3)(x - 1) = 0$ $x = 1$ of $x = -3$ $\therefore PQ = 1 + 3 = 4$ units	$\checkmark y = 0$ $\checkmark (x + 1)^2 = 4$ $\checkmark x + 1 = \pm 2$ $\checkmark PQ = 4$ units (4) $\checkmark y = 0$ $\checkmark$ standard form $\checkmark$ factors $\checkmark PQ = 4$ units (4)
20.4	$k(x) = 2(-x + 1)^2 - 8$ $= 2(x^2 - 2x + 1) - 8$ $= 2x^2 - 4x + 2 - 8$ $= 2x^2 - 4x - 6$	$\checkmark$ substituting x by $-x$ $\checkmark$ simplification $(x^2 - 2x + 1)$ $\checkmark$ answer $(2x^2 - 4x - 6)$ (3)

	OR $k(x) = 2(-x)^2 + 4(-x) - 6$ $= 2x^2 - 4x - 6$ OR $k(x) = 2(x - 1)^2 - 8$ $= 2(x^2 - 2x + 1) - 8$ $= 2x^2 - 4x + 2 - 8$ $= 2x^2 - 4x - 6$	✓ substituting x by $-x$ ✓✓ answer (3) ✓ substituting $(x + 1)$ by $(x - 1)$ ✓ simplification $(x^2 - 2x + 1)$ ✓ answer $(2x^2 - 4x - 6)$ (3)
20.5	$x = \left(\frac{1}{2}\right)^y$ $y = \log_{\frac{1}{2}} x$ OR $y = -\log_2 x$ OR $y = \log_2 \frac{1}{x}$	✓ answer (1) ✓ answer (1) ✓ answer (1)
20.6		✓ shape ✓ x-intercept ✓ point (4;-2) or any other point (3)
20.7.1	$0 < x \leq 4$ OR $x \in (0; 4]$	✓ $0 < x$ ✓ $x \leq 4$ (2) ✓✓ answer (2)
20.7.2	If $x < 0$ and $f(x) > 0$: $\therefore x < -3$ or if $x > 0$ and $f(x) < 0$: $\therefore 0 < x < 1$ OR $x \in (0; 1) \cup (-\infty; -3)$	✓✓ $x < -3$ ✓✓ $0 < x < 1$ (4) ✓✓ $(0; 1)$ ✓✓ $(-\infty; -3)$ (4)

QUESTION 21

#	SUGGESTED ANSWER	DESCRIPTORS	Ma k
21.1	OC = 6 units A; B: x-intercepts: Let $y = 0$ $\therefore -2x^2 - 4x + 6 = 0$ $\therefore x^2 + 2x - 3 = 0$ $\therefore (x + 3)(x - 1) = 0$ $\therefore x = -3$ OR $x = 1$ A(-3; 0) and B(1; 0) $\Rightarrow$ AB = 4 units	✓ OC = 6 ✓ Let $y = 0$ ✓ Factors ✓ Both x-values ✓ answer	(5)
21.2	$x = -\frac{b}{2a} = -\left[\frac{-4}{2(-2)}\right] = -1$ OR $f'(x) = -4x - 4 = 0 \Rightarrow x = -1$	✓ substitution ✓ $x = -1$ OR ✓ derivative ✓ $x = -1$ Answer only = FULL MARKS	(2)
21.3	Subst. $x = -1$ in $f(x)$ $\therefore$ ST = $-2(-1)^2 - 4(-1) + 6$ = 8 units	✓ Subst. $x = -1$ ✓ answer	(2)
21.4	$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{0-6}{-3-0} = 2$	✓ Subst. in m ✓ answer	(2)
21.5	$m_g = 2$; // lines but $m_g = f'(x) = -4x - 4 = 2$ $\therefore x = -\frac{3}{2}$ $\therefore y = -2\left(-\frac{3}{2}\right)^2 - 4\left(-\frac{3}{2}\right) + 6 = 7\frac{1}{2}$ $\Rightarrow D\left(-\frac{3}{2}; 7\frac{1}{2}\right)$	✓ $m_g = 2$ ✓ $m_g = f'(x)$ ✓ $-4x - 4 = 2$ ✓ $x = -\frac{3}{2}$ ✓ $y = 7\frac{1}{2}$	(5)
21.6	$a = -1$; the axis of symmetry OR $f(a + t) = f(a - t)$ $\therefore -2(a + t)^2 - 4(a + t) + 6$ $= -2(a - t)^2 - 4(a - t) + 6$ $\therefore -2a^2 - 4at - 2t^2 - 4a - 4t + 6$ $= -2a^2 + 4at - 2t^2 - 4a + 4t + 6$ $\therefore 8at + 8t = 0$ $\therefore 8t(a + 1) = 0$ $\therefore t = 0$ or $a = -1$	✓✓ $a = -1$ ✓ Subst. ✓ $a = -1$	(2)
			[18]

QUESTION 22

#	SUGGESTED ANSWER	DESCRIPTORS	Ma k
22.1	$x = 1$	✓ answer	(1)
22.2	x -int: Let $y = 0$ $\frac{2+x}{x-1} = 0 \Rightarrow x = -2$ $\therefore A(-2; 0)$	✓ $y = 0$ ✓ $x = -2$	(2)
22.3	y -int: Let $x = 0$ $\frac{2+0}{0-1} = y \Rightarrow y = -2$ $\therefore B(0; -2)$ Area $\triangle AOB = \frac{1}{2} AO \times OB$ $= \frac{1}{2} (2)(2) = 2 \text{ units}^2$	✓ $B(0; -2)$ ✓ Subst. in Area formula ✓ answer	(3)
22.4	$f(x) = \frac{2+x}{x-1} = \frac{x-1+3}{x-1}$ $= \frac{x-1}{x-1} + \frac{3}{x-1}$ $= \frac{3}{x-1} + 1$	✓ $\frac{2+x}{x-1} = \frac{x-1+3}{x-1}$ ✓ Simplify to: $\frac{x-1}{x-1} + \frac{3}{x-1}$	(2)
22.5	$(3; 1)$	✓ 3 (CA from 5.2 - shift 2 units to the right) ✓ 1	(2)
			[10]

QUESTION 23

#	SUGGESTED ANSWER	DESCRIPTORS	Ma k
23.1.1	$y > -1; y \in \mathbb{R}$	✓ ✓ $y > 0; y \in \mathbb{R}$	(2)
23.1.2	$g(x) = 2^x$ $\therefore g^{-1}: y = \log_2 x$	✓ $g(x) = 2^x$ ✓ $y = \log_2 x$	(2)
23.2.1	$k(x) = 3x^2; x \leq 0$	✓ $k(x) = 3x^2$ ✓ $x \leq 0$	(2)
23.2.2	$(0; 0)$ OR origin	✓ ✓ Answer	(2)
			[8]